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M. G. Crandall completed his program ~~of~~ obtaining the basic results of Kato's theory of quasilinear evolution equations by the simpler methods of nonlinear semigroup theory (implicit differencing in time), in a joint work with P. Souganidis. He studied, with P. L. Lions and P. Souganidis, certain classes of parabolic and Hamilton-Jacobi equations, showing the existence and uniqueness of solutions in initial-boundary value problems with singular (e.g. identically infinite) initial data and the condinuous dependence of these singular solutions as the diffusion coefficient tends to zero. This work shows how certain pde questions motivated by the theory of large deviations can be treated in greater generality and provides a certain abstract framework for this as well as concrete estimates. (jlo)

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FINAL TECHNICAL REPORT

AFOSR-87-0202

Michael G. Crandall and Paul H. Rabinowitz, Principal Investigators

June 1, 1987 through January 31, 1989

AFOSR-TR- 89-0918

Principal Investigators:

M. Crandall completed his program of obtaining the basic results of Kato's theory of quasilinear evolution equations by the simpler methods of nonlinear semigroup theory (implicit differencing in time) in a joint work with P. Souganidis. He studied, with P. L. Lions and P. Souganidis, certain classes of parabolic and Hamilton-Jacobi equations, showing the existence and uniqueness of solutions of initial-boundary value problems with singular (e.g., identically infinite) initial-data and the continuous dependence of these singular solutions as the diffusion coefficient tends to zero. This work shows how certain pde questions motivated by the theory of large deviations can be treated in greater generality and provides a certain abstract framework for this as well as concrete estimates.

Another work, with R. Newcomb and Y. Tomita, extends some previous results on growth classes for Hamilton-Jacobi equations in $\mathbf{R}^n$ to the case of degenerate elliptic problems in $\mathbf{R}^n$. He completed, with P. L. Lions, the first major work on treating Hamilton-Jacobi equations in which the Hamiltonians involve semigroup generators by viscosity methods, thereby beginning the extension of viscosity theory to a generality which can justify the dynamic programming equation for the value function as it arises in the control of pdes. Finally, he obtained a significant simplification of the basic result used to study the uniqueness of generalized (viscosity) solutions of fully nonlinear elliptic pdes.

Publications:

[1] (with P. Souganidis) On nonlinear evolution equations, to appear in Nonlin. Anal. Th. Meth. Appl. (ms. 26 pages).

[2] (with P. L. Lions and P. Souganidis) Maximal solutions and universal bounds for some quasilinear evolution equations of parabolic type, Arch. Rat. Mech. Anal. 105 (1989), 163–190.

[3] (with R. Newcomb and Y. Tomita) Existence and uniqueness of viscosity solutions of degenerate quasilinear elliptic equations in $\mathbf{R}^n$, Applicable Analysis, to appear (ms. 19 pages).

[4] (with P. L. Lions) Viscosity solutions of Hamilton-Jacobi equations in infinite dimensions. Part IV. Hamiltonians with unbounded linear terms, J. Func. Anal., to appear (ms. 50 pages).

[5] Semidifferentials, quadratic forms and viscosity solutions of fully nonlinear elliptic equations, Ann. I.H.P. Anal. Non. Lin., to appear (ms. 18 pages).

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P. Rabinowitz studied several questions involving the application of variational methods to Hamiltonian systems of ordinary differential equations. In [1], he studied forced second order Hamiltonian systems

$$(*) \qquad \ddot{q} + V_q(t,q) = f(t)$$

where V and f are periodic in T and V is periodic in the components $q_1, \ldots, q_n$ of q. He established the existence of at least $n+1$ distinct time periodic solutions of $(*)$ when f is of mean value 0. Later in [4], he treated $(*)$ when $f \equiv 0$ and V is independent of t, in which case $(*)$ has at least $n+1$ equilibrium solutions. He showed that if the set of maxima of V are isolated, there are heteroclinic orbits emanating from each such maximum of V.

In another direction, in joint work with A. Bahri [2], minimax methods were developed to establish the existence of periodic solutions of $(*)$ when V is singular. The singularities treated in [2] are point singularities such as arise in the Kepler problem. A notion of generalized periodic solution was introduced which should be useful in singular problems whenever collision orbits are possible. Finally in [3], some of the ideas of [2] were extended to obtain subharmonic solutions for singular problems and recent topological results of Fadell and Husseini were used to treat a more general class of forced singularities than the techniques of [2] could handle.

Publications:

[1] On a class of functionals invariant under a $\mathbb{Z}^n$ action, Trans. A.M.S. **310** (1988), 303–311.

[2] (with A. Bahri) A minimax method for a class of Hamiltonian systems with singular potentials, J. Functional Anal. **82** (1989), 412–428.

[3] Periodic soltuions for some forced singular Hamiltonian systems, to appear, Festschrift in Honor of Jürgen Moser, Academic Press.

[4] Periodic and heteroclinic orbits for a periodic Hamiltonian system, to appear, Analyse Nonlineaire.

Postdoctoral Support:

Sigurd Angenent has been working in three main directions:

(a) Questions related to the smoothness of the free boundary for the porous medium equation.

(b) Monotone twist maps, an area of much current interest in dynamical systems. He has introduced new approaches to this subject using discrete versions of the maximum principle. This work has resulted in simpler ways of proving difficult results as well as the resolution of hitherto intractable questions.

(c) The evolution of curves on surfaces, a subject initiated by geometers. He has reformulated the problem in a way more amenable to attack by PDE methods resulting in the simplification of old results and the discovery of new ones. Also, jointly with M. Gurtin, he has expanded this study to phase transition problems, like the melting of solids, which have a related but more complicated mathematical structure.

Publications:

[1] Monotone recurrence relations, then Birkhoff orbits and topological entropy, to appear, J. Ergotic Th. and Dynamical Systems.

[2] Nodal properties of solutions of parabolic equations, to appear, Proc. Conf. on Nonlinear P.D.E., Provo, Utah.

[3] Parabolic equations for curves on surfaces, I: Curves with p-integrable curvature, preprint.

[4] Parabolic equations for curves on surfaces, II: Intersections, blowup and the global flow, preprint.

[5] (with M. Gurtin) Multiphase thermodynamics with interfacial structure: 2. Evolution of an isothermal interface, preprint.

[6] Solutions of the $1-D$ porous medium equation are determined by their free boundary, preprint.

Research Assistants:

The people supported here were Y. Long, R. Newcomb, E. Silva, P. Felmer, and S. P. Yung. Long, Newcomb, and Silva completed their Ph.D. theses.

Long showed that certain classes of periodically forced abstract Hamiltonian systems – including certain classes of Hamiltonian systems of ordinary differential equations, nonlinear Schroedinger equations, and nonlinear wave equations – have dense range. He also proved that certain autonomous Hamiltonian systems, which have infinitely many periodic solutions, when subjected to a forced perturbation (which need not be small) also have the same multiplicity of solutions. His thesis was entitled "Periodic Solutions of Perturbed Superquadratic Hamiltonian Systems".

Newcomb produced a thesis on the connection between the general existence and uniqueness theory for *viscosity solutions* of Hamilton-Jacobi equations and the theory of differential games. In the process he verified the principle of dynamic programming in a new generality. His thesis was entitled "Existence and Correspondence of Value Functions and Viscosity Solutions of Hamilton-Jacobi Equations".

Silva worked on critical point theorems in an abstract setting, mainly of saddle point type and found new examples of topological linking phenomena. He also gave many applications to nonlinear elliptic partial differential equations and to periodic solutions of Hamiltonian systems. His thesis is called "Critical Point Theorems and Applications to Differential Equations".

Felmer made progress towards his Ph.D. He will finish in August, 1989. His research concerns generalizations of the Birkhoff-Lewis Theorem on the existence of subharmonic solutions of Hamiltonian systems. He also has results on the existence of multiple periodic solutions of compound pendulum type systems of differential equations.

Lastly, Yung has begun work on his dissertation.